11.8

FIND THE RADIUS + INTERVAL OF CONVERGENCE

FOR $\displaystyle\sum_{n=0}^{\infty} \frac{(x+6)^n}{\sqrt{n+1}}$ CENTERED ABOUT $x=-6$

RATIO TEST

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \boxed{\lim_{n\to\infty}\left|\frac{(x+6)^{n+1}}{\sqrt{n+2}}\cdot\frac{\sqrt{n+1}}{(x+6)^n}\right|}$$

$$= \lim_{n\to\infty}\left|(x+6)\sqrt{\frac{n+1}{n+2}}\right|$$

$$= \boxed{|x+6|\lim_{n\to\infty}\sqrt{\frac{1+\frac{1}{n}}{1+\frac{2}{n}}}}$$

$$= |x+6|\cdot 1$$

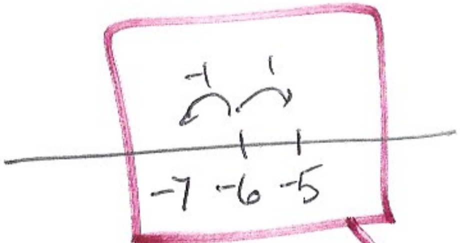
$$= \boxed{|x+6|}\boxed{<1}$$ FOR CONV

RADIUS OF CONV $|x-a|<R$

$$-1 < x+6 < 1$$

$$\boxed{-7 < x < -5}$$

OR



CONV FOR $x \in (-7,-5)$

BUT WHAT ABOUT $x=-7$ AND $x=-5$?

$x = -7$

$$\sum_{n=0}^{\infty} \frac{(-7+6)^n}{\sqrt{n+1}} = \boxed{\sum_{n=0}^{\infty} \frac{(-1)^n}{\sqrt{n+1}}} \quad \boxed{\text{CONV (ALT)}}$$

ALT SIGNS

$$\boxed{\frac{1}{\sqrt{n+1}} > 0}$$

$$f(x) = (x+1)^{-\frac{1}{2}}$$

$$f'(x) = -\frac{1}{2}\underbrace{(x+1)^{-\frac{3}{2}}}_{>0} < 0$$

$\frac{1}{\sqrt{n+1}}$ DECR

$$\boxed{\lim_{n\to\infty} \frac{1}{\sqrt{n+1}} = 0}$$

$x = -5$

$$\sum_{n=0}^{\infty} \frac{(-5+6)^n}{\sqrt{n+1}} = \boxed{\sum_{n=0}^{\infty} \frac{1}{\sqrt{n+1}}}$$

$0 < n+1 < 4n$ FOR $n \geq 1$

$0 < \sqrt{n+1} < \sqrt{4n} = 2\sqrt{n}$ $(1 < 3n)$ $(n > \frac{1}{3})$

$$\boxed{\frac{1}{\sqrt{n+1}} > \frac{1}{2\sqrt{n}} > 0}$$

$\frac{1}{2}\sum_{n=0}^{\infty} \frac{1}{n^{\frac{1}{2}}}$ DIV (p-SERIES) $\quad p = \frac{1}{2} \leq 1$

$\sum_{n=0}^{\infty} \frac{1}{\sqrt{n+1}}$ DIV (COMP)

SO, $\sum_{n=0}^{\infty} \frac{(x+6)^n}{\sqrt{n+1}}$ CONV ON $\boxed{[-7, -5)}$

$\boxed{\text{RADIUS} = 1}$ $\quad \frac{-5 - -7}{2} = 1$

11.9

$$\sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + x^4 + \cdots = \frac{1}{1-x}$$

$*x \quad *x \quad *x \quad *x$

CENTERED ABOUT $x=0$

GEO SERIES

$r = x$

$S = \frac{a_1}{1-r}$

RADIUS OF CONV = 1

CONV ON $(-1, 1)$

$-1 < x < 1$

$$\boxed{\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n}$$

$$\text{eg. } \frac{3}{1+2x} = 3 \cdot \frac{1}{1-(-2x)} = 3\sum_{n=0}^{\infty} (-2x)^n = \sum_{n=0}^{\infty} 3(-2x)^n$$

SUBSTITUTE $-2x$ FOR x IN

$$= \sum_{n=0}^{\infty} 3(-2)^n x^n$$

INTERVAL OF CONV

$-1 < -2x < 1$

$\frac{1}{2} \cancel{<} x \cancel{<} -\frac{1}{2}$

$-\frac{1}{2} < x < \frac{1}{2}$ $\quad \left(-\frac{1}{2}, \frac{1}{2}\right) \quad R = \frac{1}{2}$

eg. $\dfrac{1}{4-x} = \boxed{\dfrac{1}{4(1-\frac{1}{4}x)}} = \boxed{\sum_{n=0}^{\infty} \frac{1}{4}\left(\frac{1}{4}x\right)^n} = \boxed{\sum_{n=0}^{\infty} \frac{1}{4^{n+1}} \cdot x^n \text{ or } \sum_{n=0}^{\infty} \frac{x^n}{4^{n+1}}}$

SUB $\frac{1}{4}x$ FOR x IN Σ

INTERVAL OF CONV $= \boxed{(-4,4)}$

$$\boxed{-1 < \frac{1}{4}x < 1}$$

$$-4 < x < 4$$

eg. $\dfrac{5}{2x+3} = \dfrac{5}{3} \dfrac{1}{1-(-\frac{2}{3}x)} = \displaystyle\sum_{n=0}^{\infty} \frac{5}{3}\left(-\frac{2}{3}x\right)^n$

$$= \sum_{n=0}^{\infty} \frac{5}{3}\left(-\frac{2}{3}\right)^n x^n \text{ OR } \sum_{n=0}^{\infty} \frac{5(-2)^n}{3^{n+1}} x^n$$

$$-1 < -\frac{2}{3}x < 1$$

$-\frac{3}{2} < x < \frac{3}{2}$ CONV ON $\left(-\frac{3}{2}, \frac{3}{2}\right)$